Magnetic Circuits

EE 340

Spring 2013

Ampere's Law

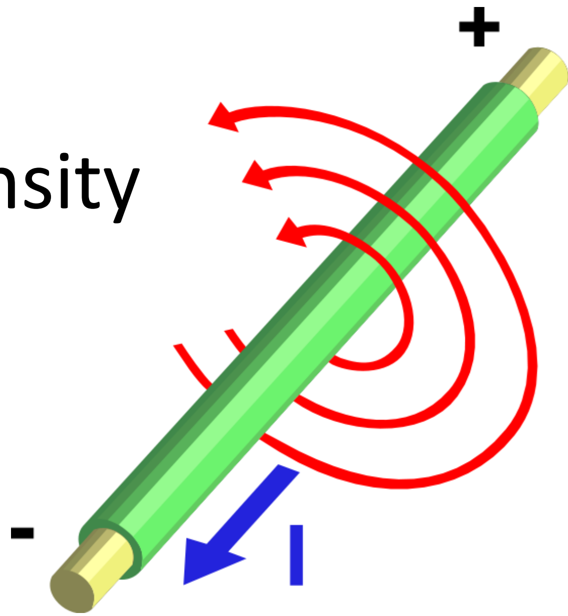
- Ampère's circuital law, discovered by André-Marie Ampère in 1826, relates the integrated magnetic field around a closed loop to the electric current passing through the loop.

$$\oint H.dl = I$$

where H is the magnetic field intensity

- At a distance *r* from the wire,

$$\oint H.dl = H.(2\pi r) = I$$



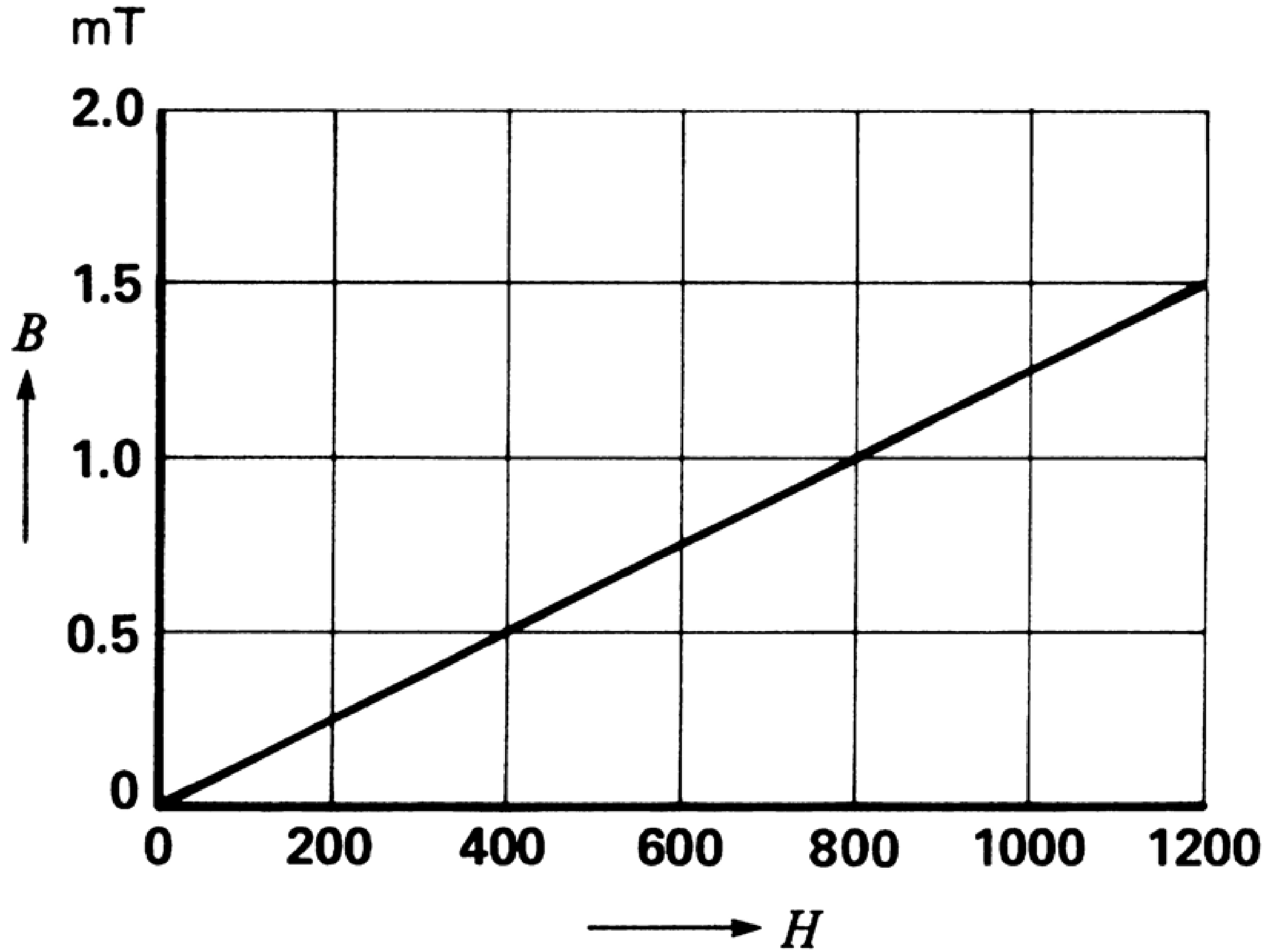
Magnetic Flux Density

- Relation between magnetic field intensity H and magnetic field density B:

$$B = \mu H = (\mu_r \mu_0) H$$

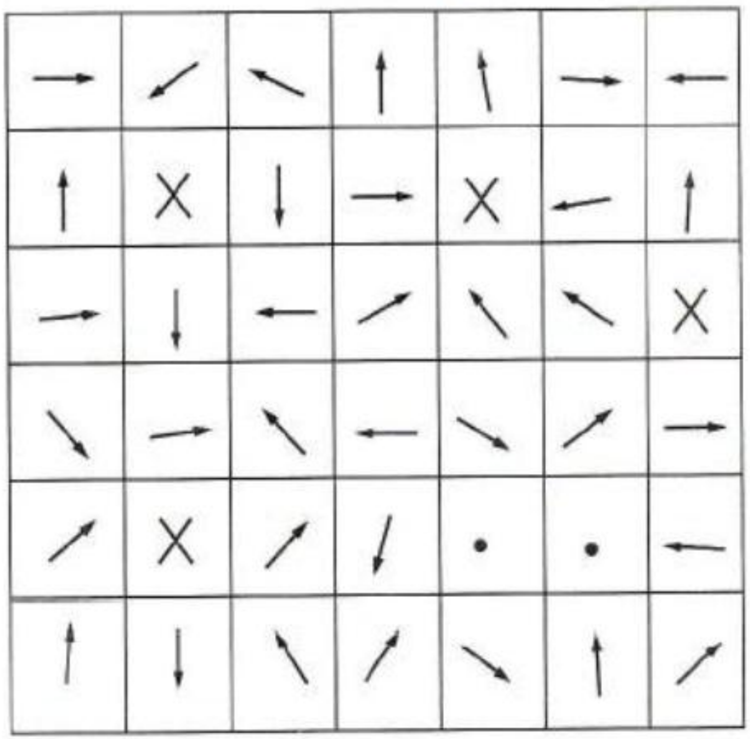
where is μ_r is the relative permeability of the medium (unit-less), is μ_o is the permeability of free space ($4\pi x 10^{-7}$ H/m)

B-H Curve in air and non-ferromagnetic material

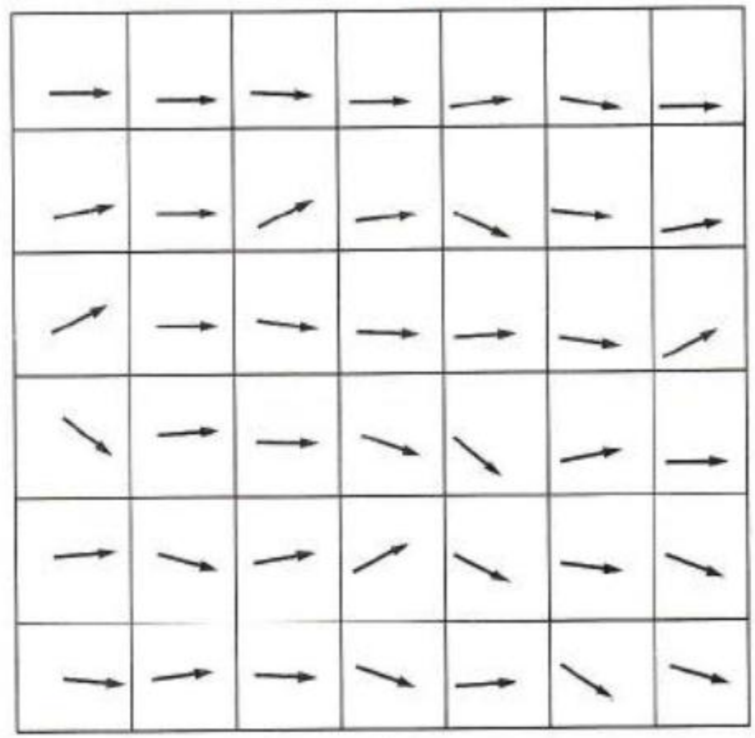


Orientation of magnetic domains without and with the presence of an external magnetic field

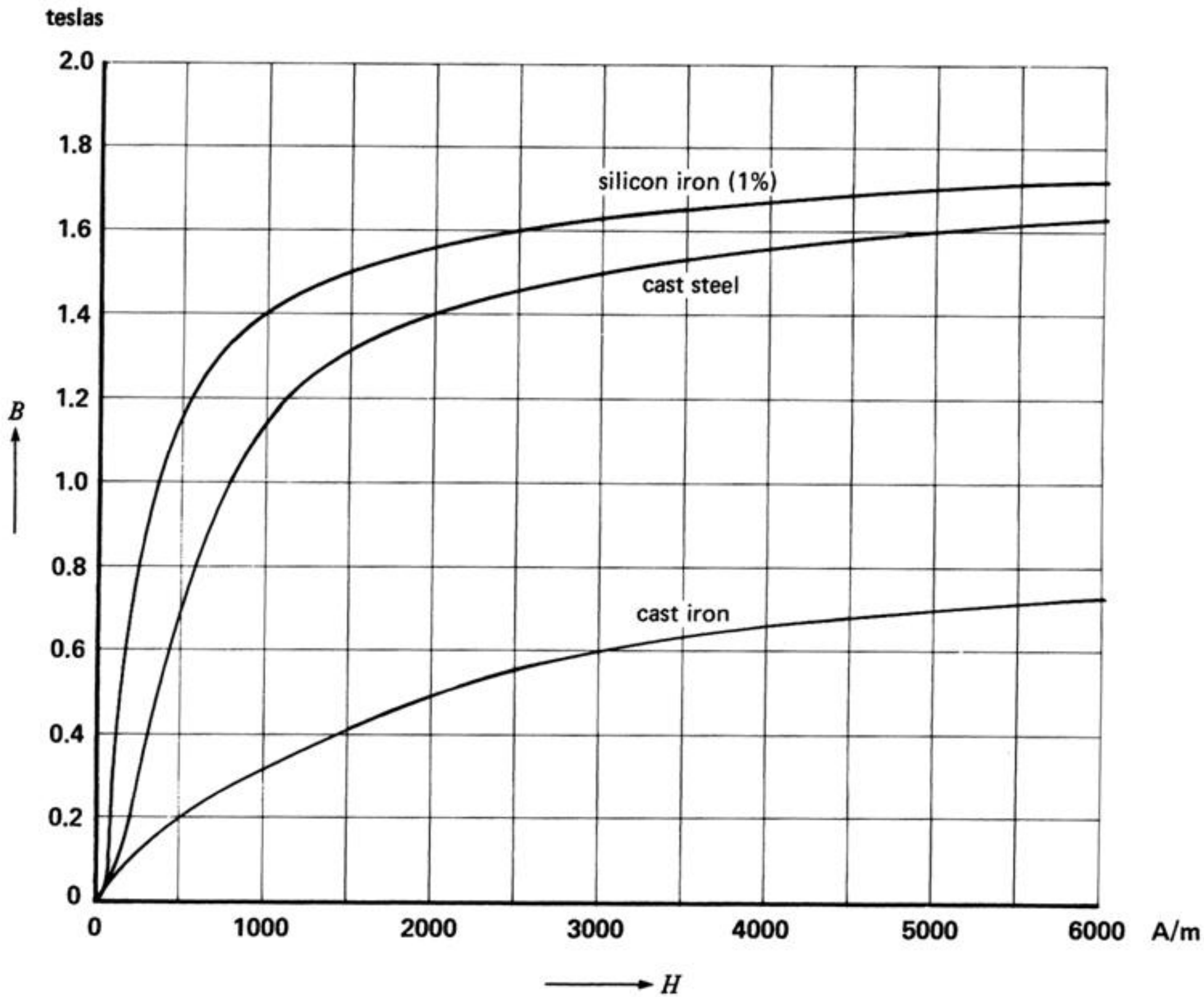
Without external magnetic field



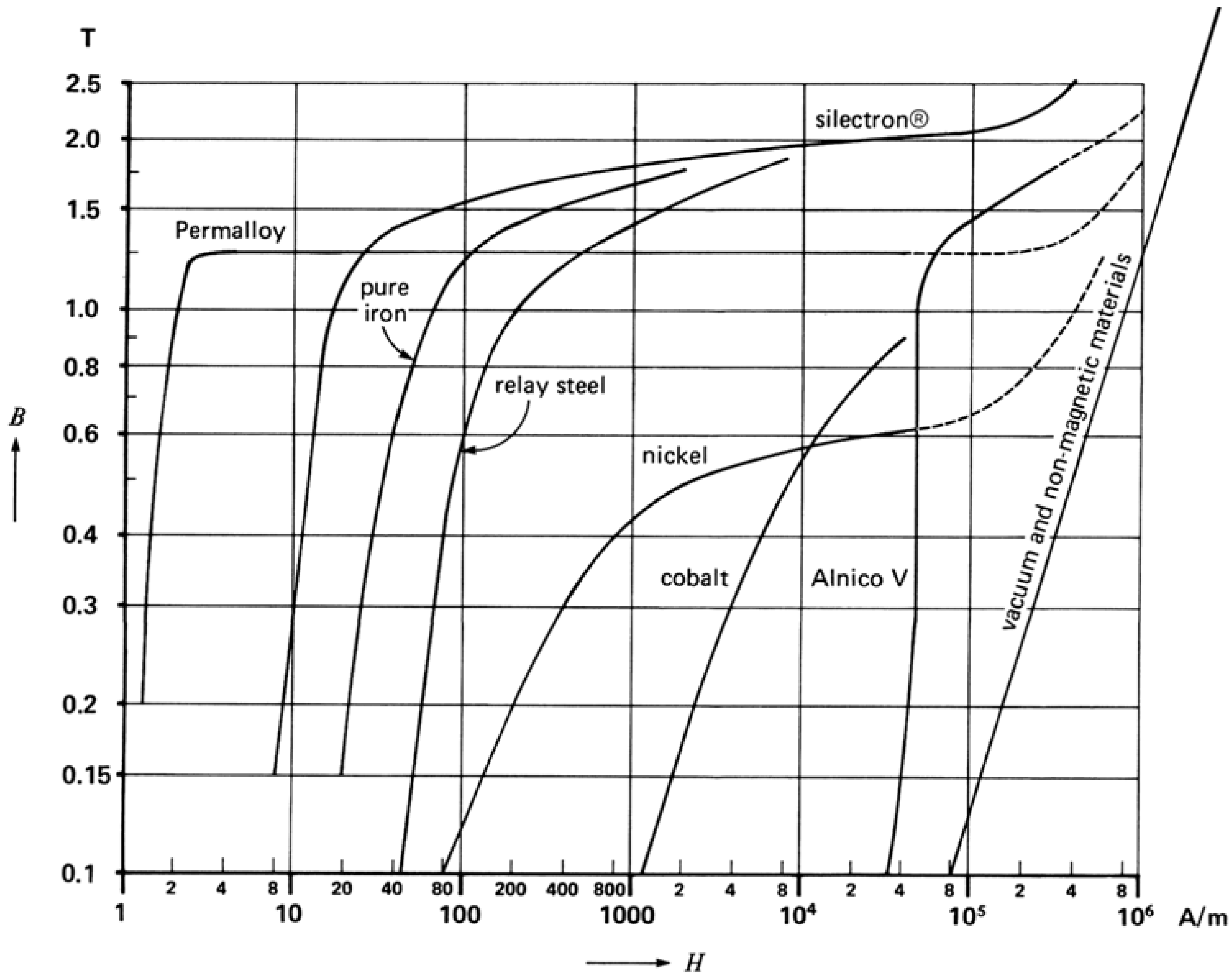
With external magnetic fiedl



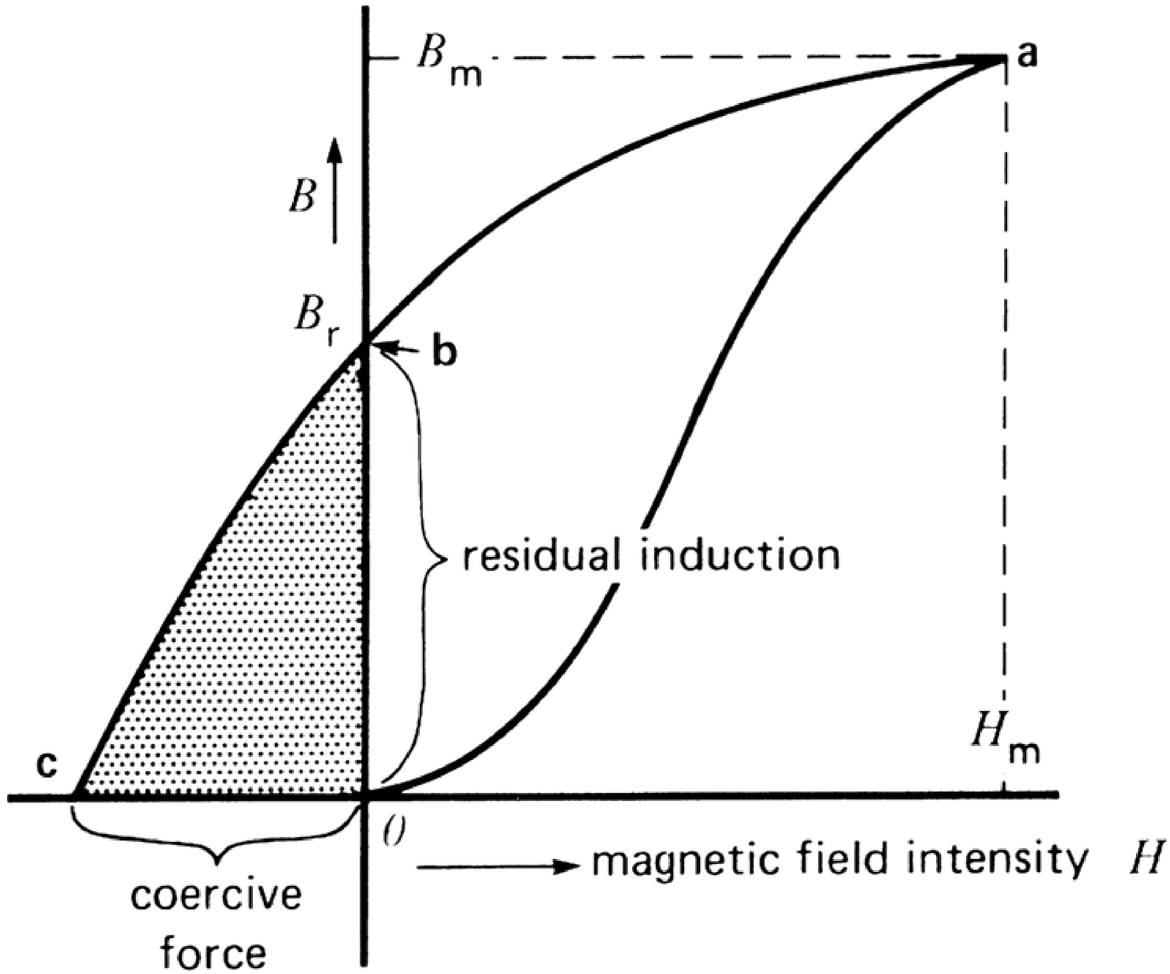
B-H Curve in 3 Ferromagnetic materials



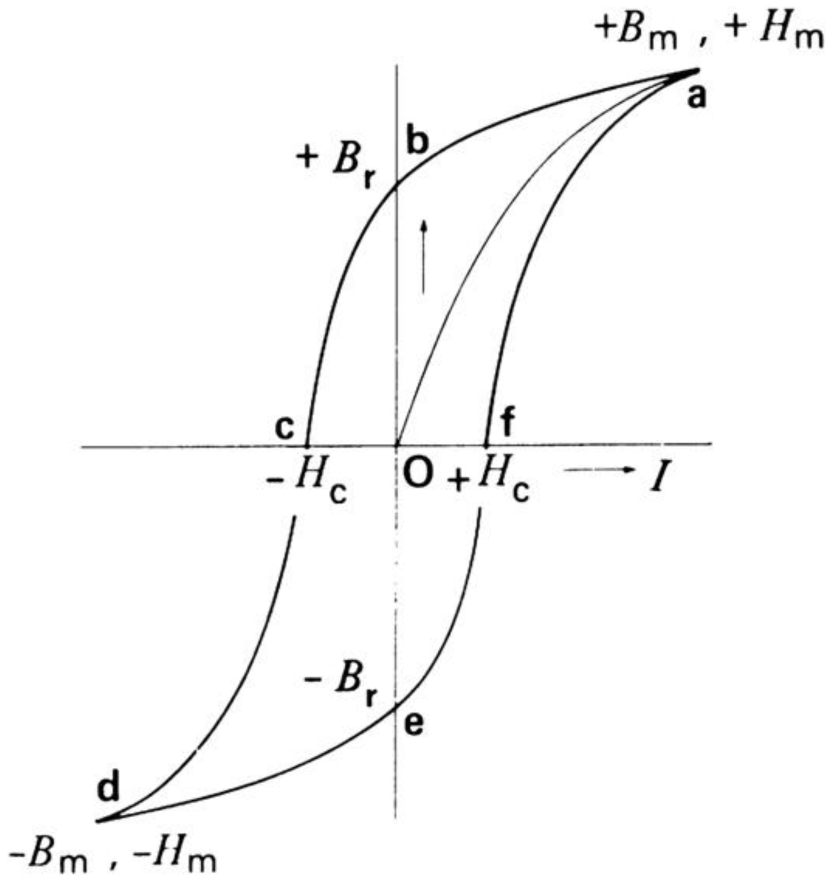
Saturation curves of magnetic and nonmagnetic materials



Residual induction and Coercive Force



Hysteresis Loop (AC Current)



Magnetic Flux

- Magnetic flux is the total flux within a given area. It is obtained by integrating the flux density over this area:

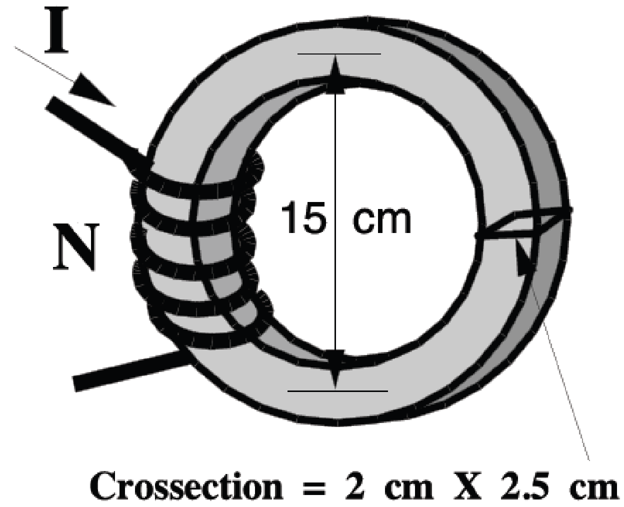
$$\phi = \int BdA$$

- If the flux density is constant throughout the area, then,

$$\phi = BA$$

Ampere's Law applied to a magnetic circuit (Solid Core)

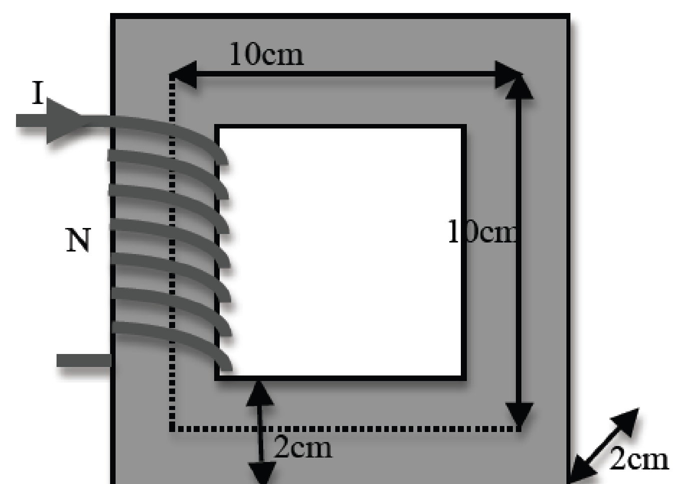
$$\oint H.dl = Hl = \frac{B}{\mu} l = NI$$



- Magnetic flux (Wb):

$$\phi = \int Bds = BA$$

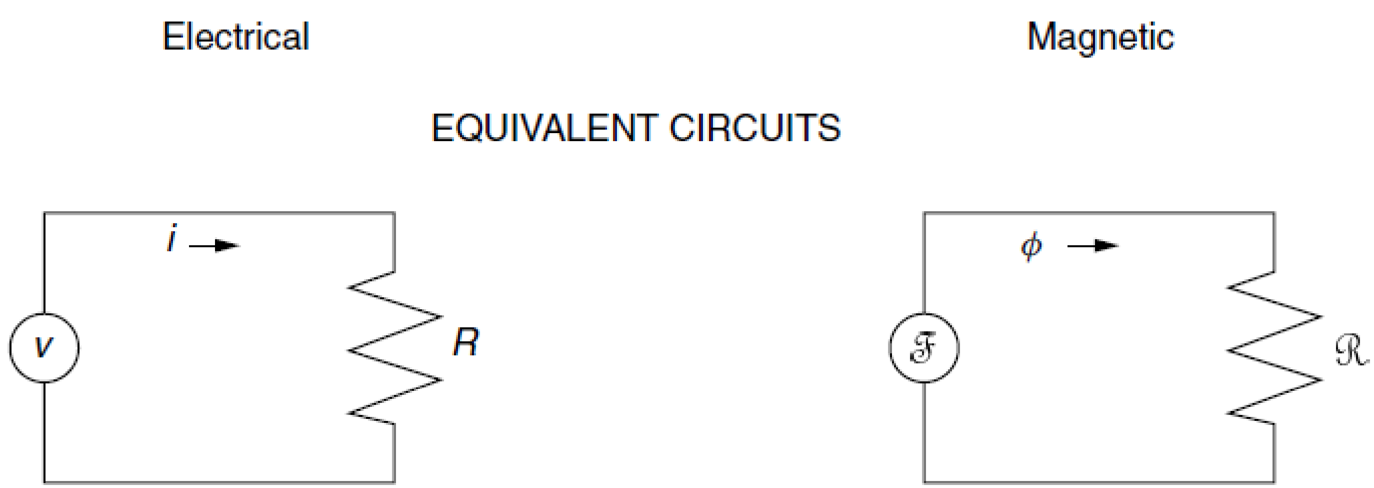
- Hence, $$NI = \phi\left(\frac{l}{\mu A}\right)$$



Analogy between electric and magnetic circuits

TABLE 1.4 Analogous Electrical and Magnetic Circuit Quantities

Electrical	Magnetic	Magnetic Units
Voltage v	Magnetomotive force $\mathcal{F} = Ni$	Amp-turns
Current i	Magnetic flux ϕ	Webers Wb
Resistance R	Reluctance $\mathcal{R}$	Amp-turns/Wb
Conductivity $1/\rho$	Permeability μ	Wb/A-t-m
Current density J	Magnetic flux density B	$\mathrm{Wb/m^2}$ = teslas T
Electric field E	Magnetic field intensity H	Amp-turn/m



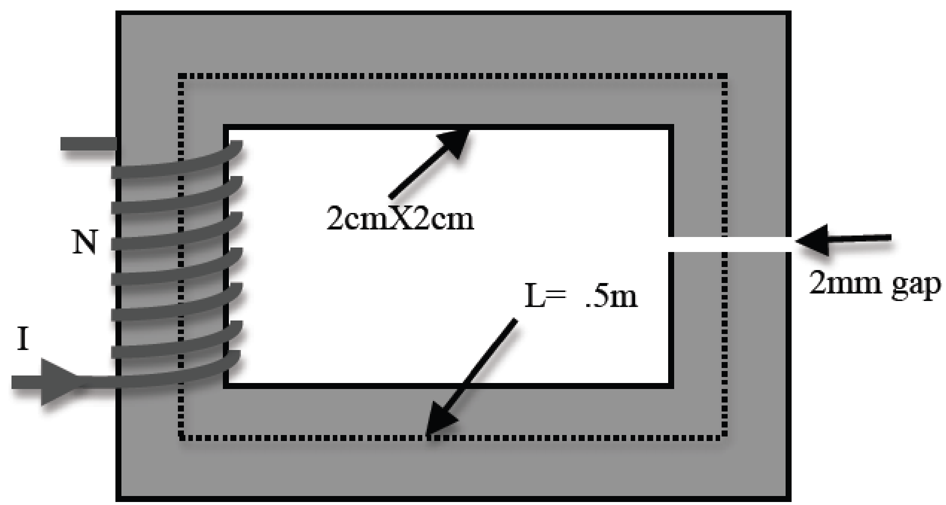
Ampere's Law applied to a magnetic circuit (core with air gap)

$$\oint H.dl = H_c l_c + H_a l_a = \frac{B}{\mu_r \mu_o} l_c + \frac{B}{\mu_o} l_a = NI$$

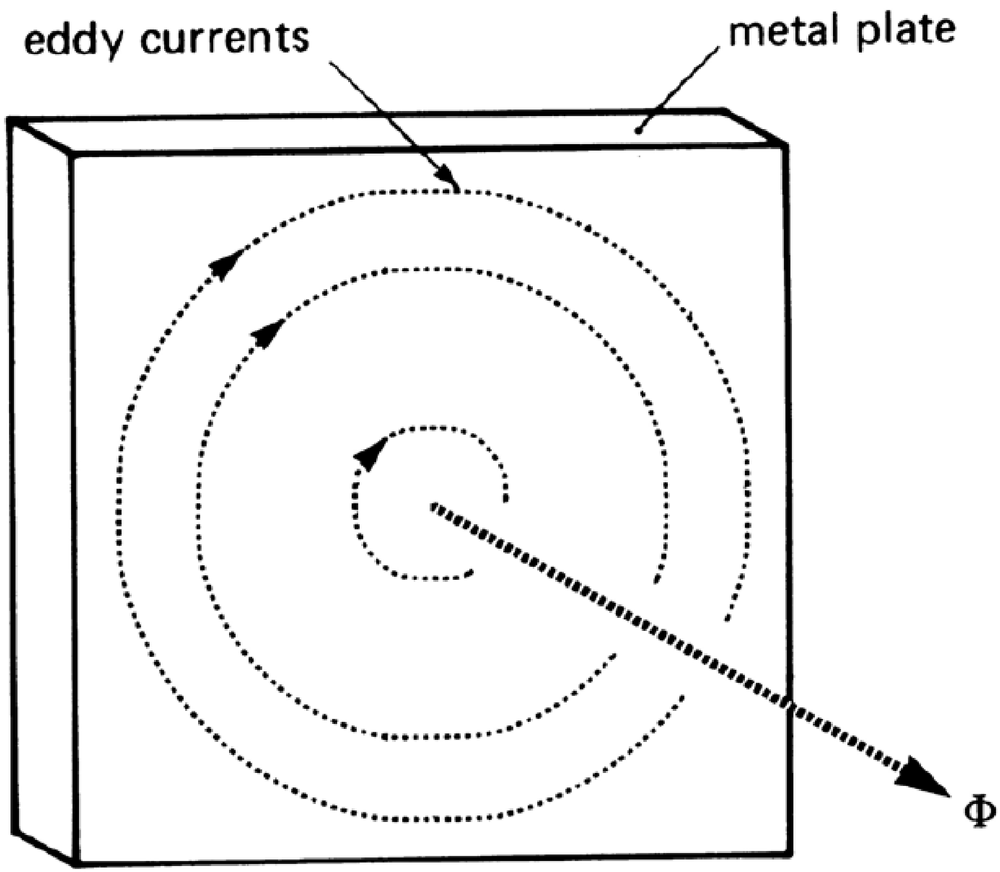
$$NI = \phi \Re$$

where

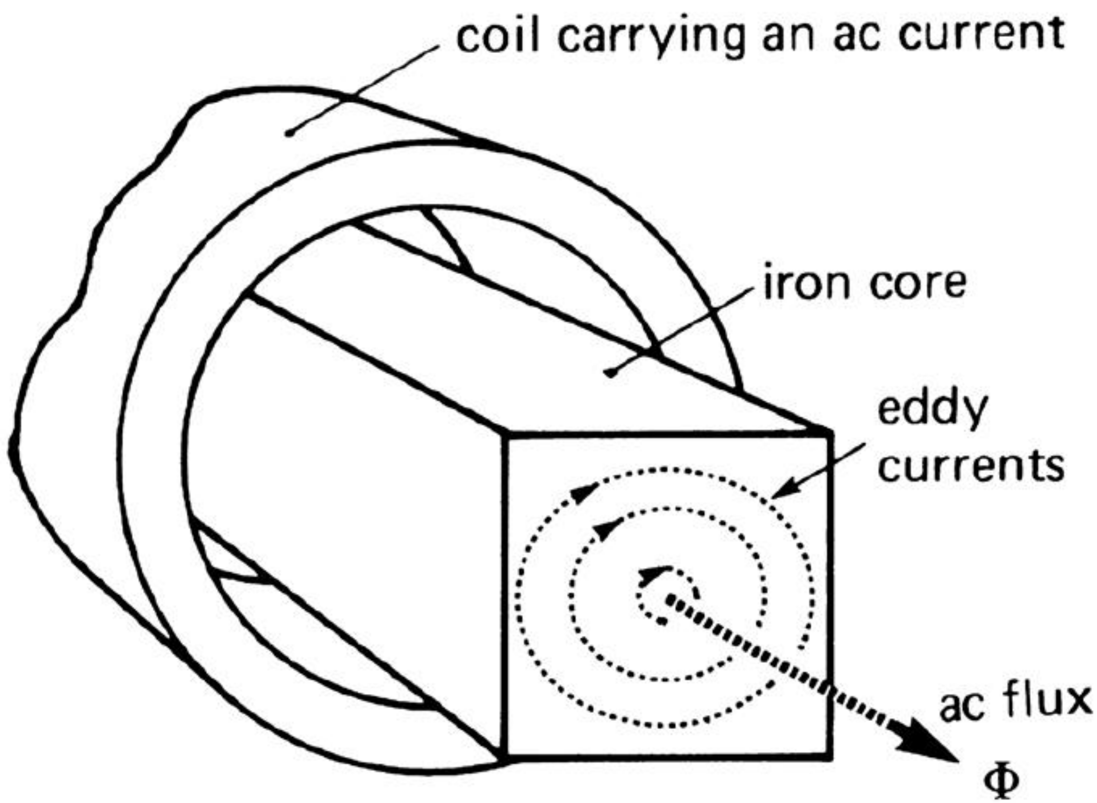
$$\Re = \left(\frac{l_c}{\mu_r \mu_o A} + \frac{l_a}{\mu_o A} \right)$$



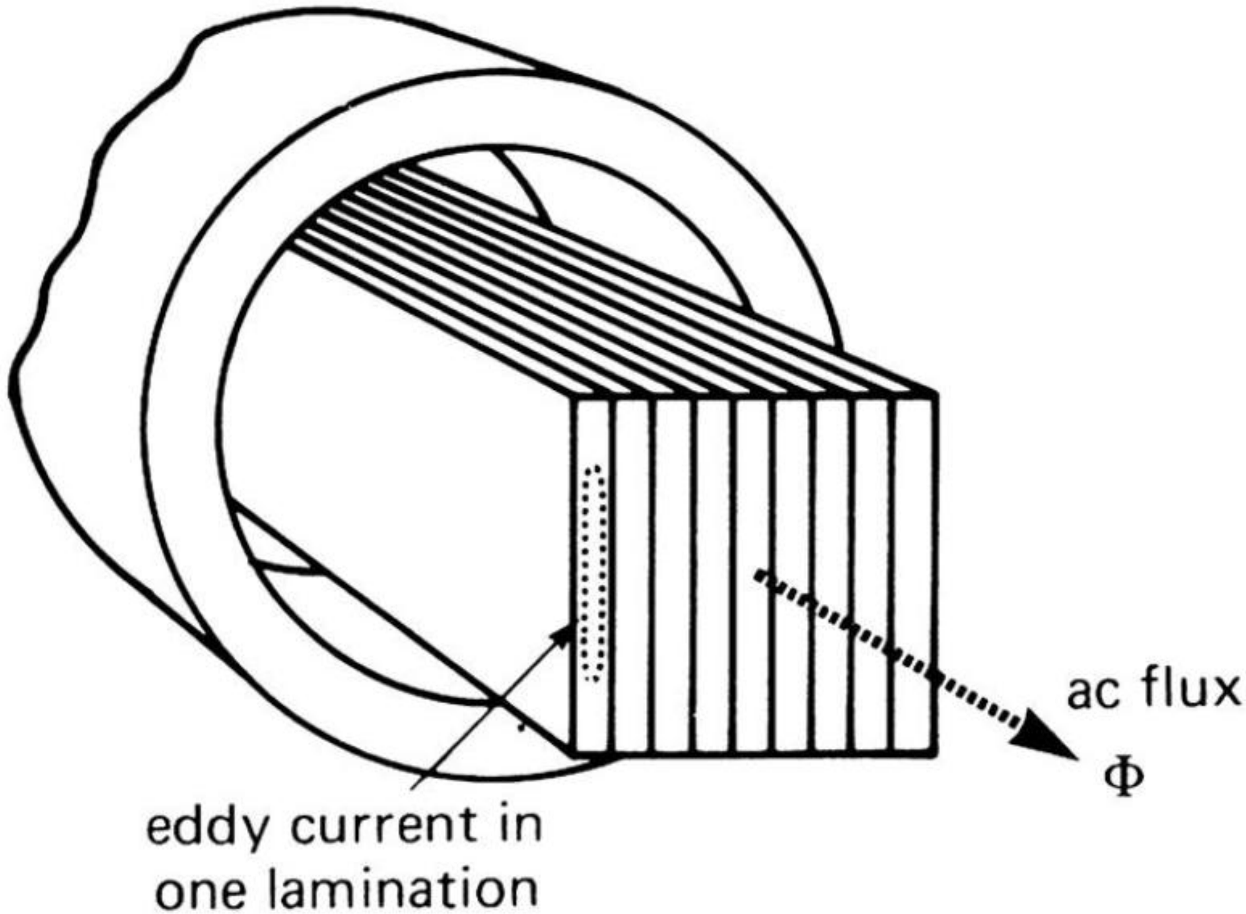
Eddy currents are induced in a solid metal plate under the presence of a varying magnetic field



Solid iron core carrying an AC flux (significant eddy current flow)



Core built up of insulated laminations minimizes eddy currents (and eddy current losses)



Faraday’s Law

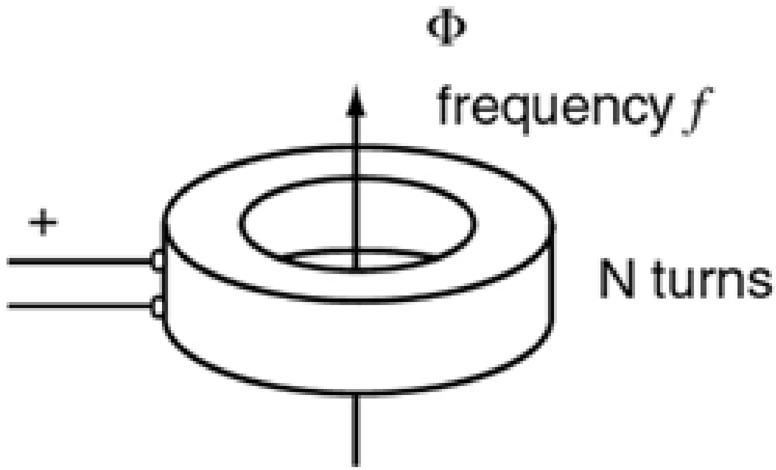
- Faraday's law of induction is a basic law of electromagnetism relating to the operating principles of transformers, inductors, electrical motors and generators. The law states that:

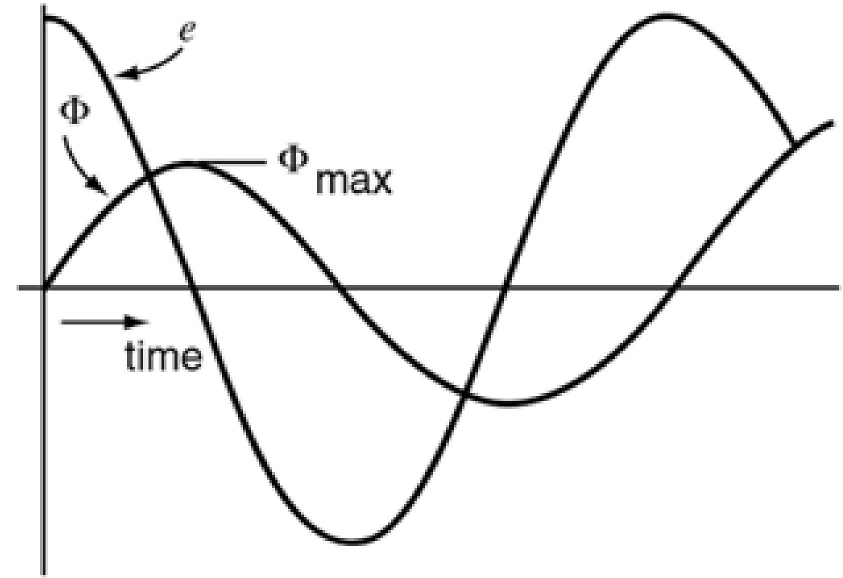
“The induced electromotive force (EMF) in any closed circuit is proportional to the time rate of change of the magnetic flux through the circuit”

Or alternatively, “the EMF generated is proportional to the rate of change of the magnetic flux”.

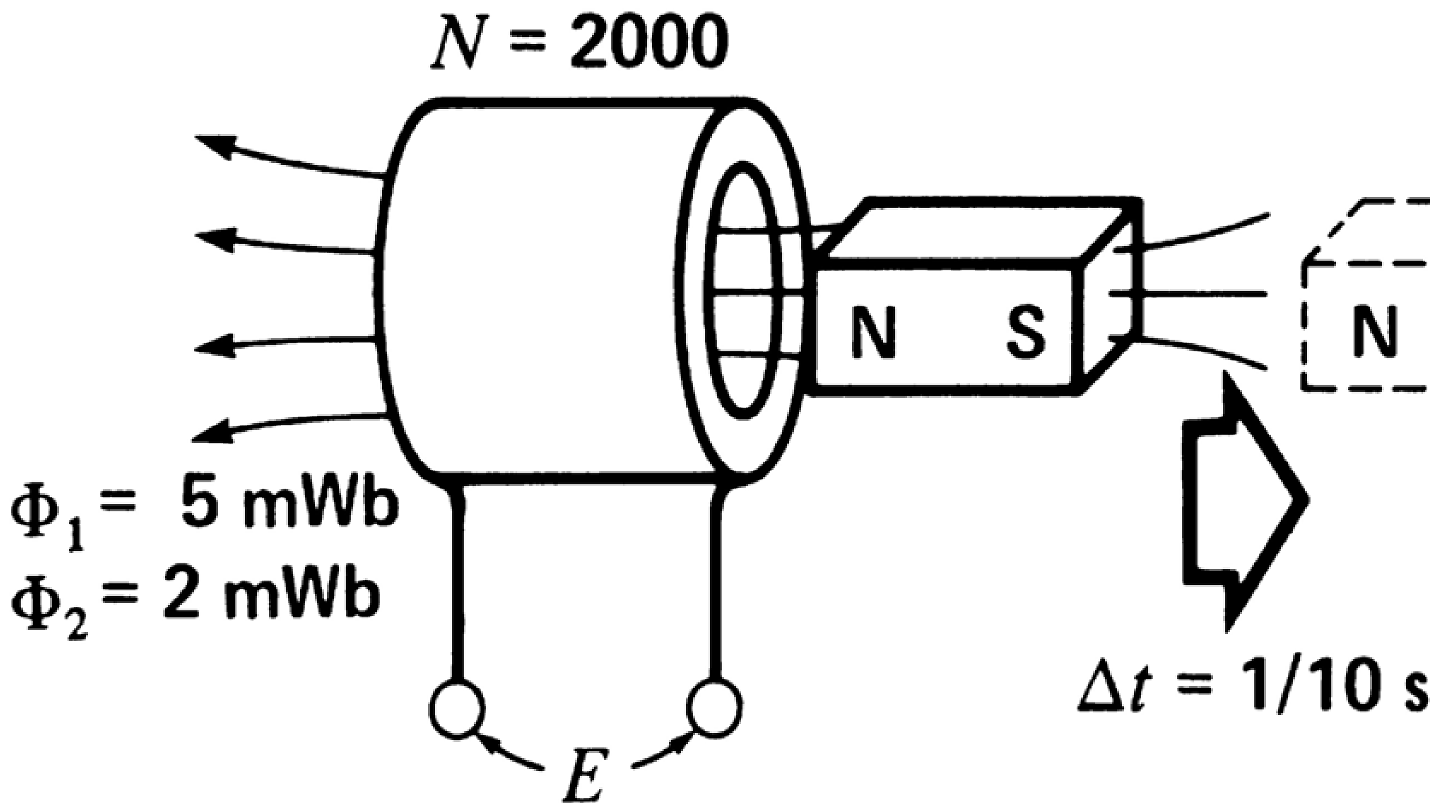
$$e = -N\frac{d\phi}{dt}$$

Voltage induced in a coil when it links a variable flux in the form of a sinusoid





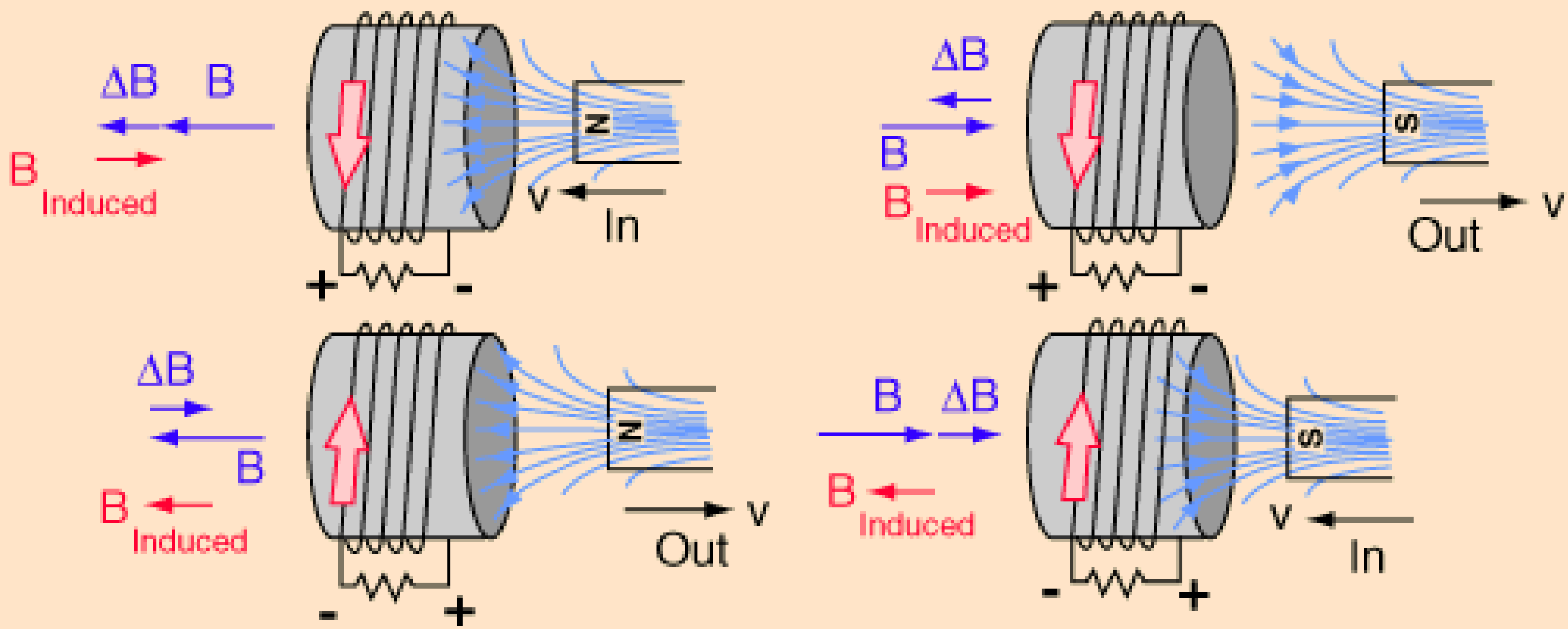
Example: voltage induced in a coil by a moving magnet



$E = -N\Delta\varphi/\Delta t = -2000(-3/0.1) = 60{,}000$ mV or 60 V

Lenz's Law

When an emf is generated by a change in magnetic flux according to Faraday's Law, the polarity of the induced emf is such that it produces a current whose magnetic field opposes the change which produces it. The induced magnetic field inside any loop of wire always acts to keep the magnetic flux in the loop constant. In the examples below, if the B field is increasing, the induced field acts in opposition to it. If it is decreasing, the induced field acts in the direction of the applied field to try to keep it constant.

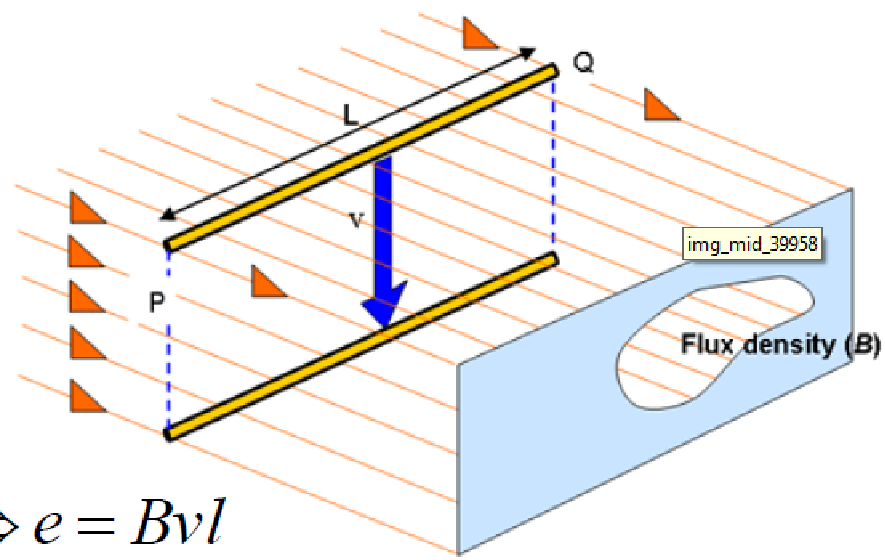


Induced voltage in a conductor moving in a magnetic field

- The voltage induced in a conductor of length *l* that is moving in a magnetic field with flux density *B*, at a speed *v* is given by

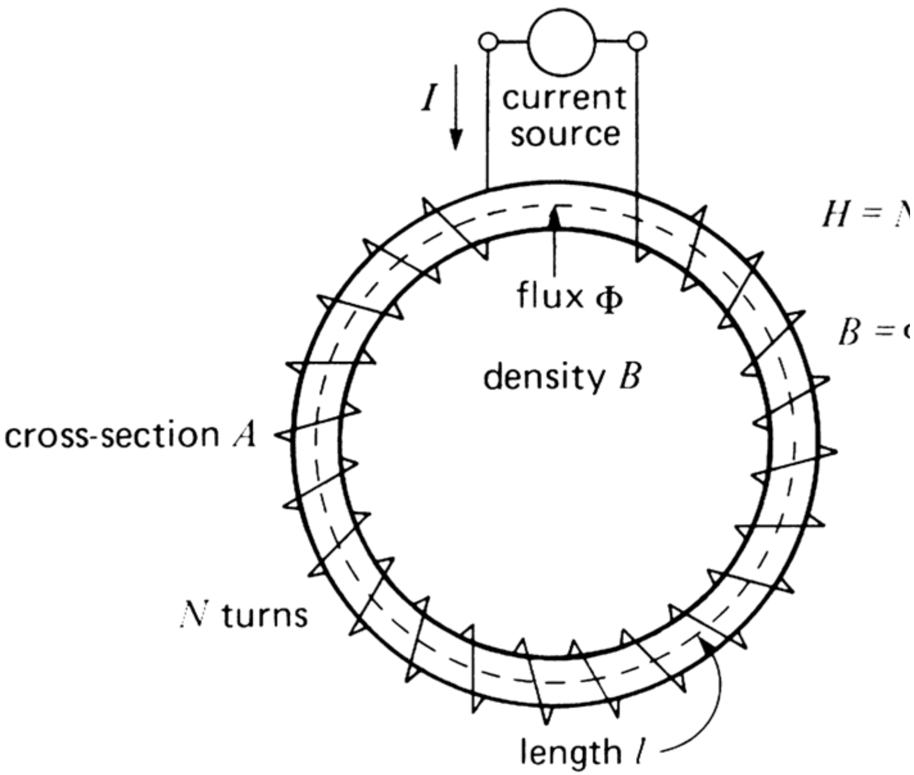
$$e = (vB\sin\theta)l\cos\phi$$

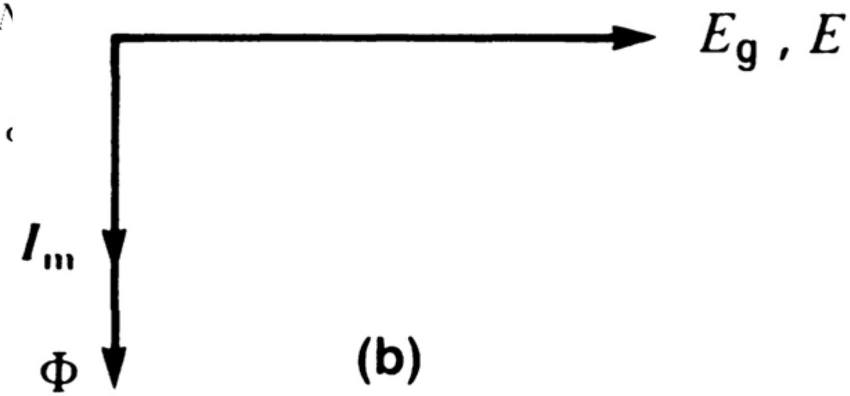
where θ is the angle between vxB and the velocity vector, and ϕ is the angle between vxB and the wire. The polarity of the induced voltage is determined by Lenz's Law.



$$\theta = 90\deg. \quad and \quad \phi = 0\deg \Rightarrow e = Bvl$$

Inductance of a coil





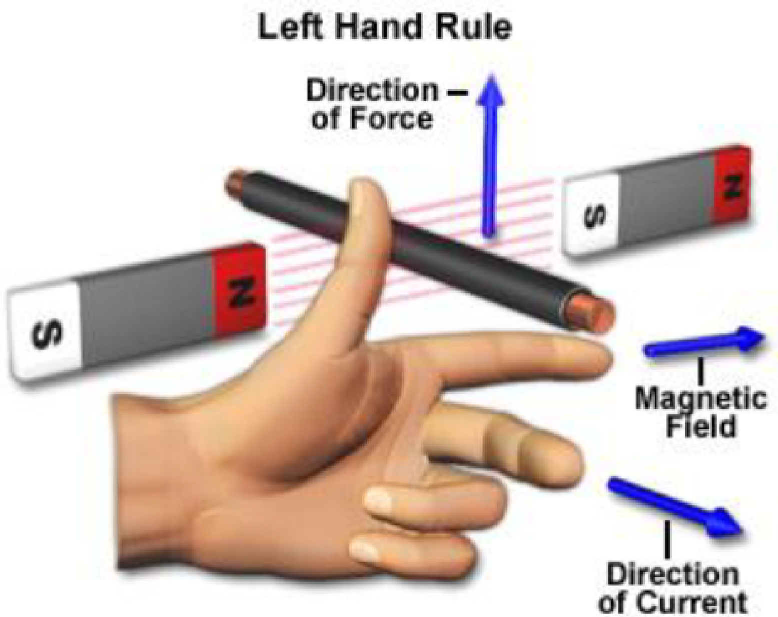
$$e = L\frac{di}{dt} = N\frac{d\phi}{dt} = N\frac{d(Ni\mu A/l)}{dt} = (N^2\mu A/l)\frac{di}{dt} \longrightarrow L = \frac{N^2\mu A}{l}$$

Induced force on a current-carrying conductor

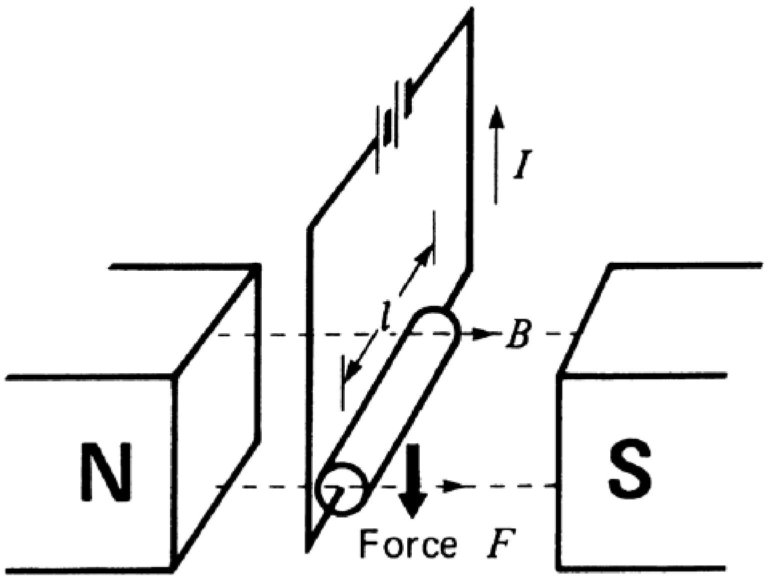
- The force on a wire of length *l* and carrying a current *i* under the presence of a magnetic flux B is given by

$$F = Bil \sin\theta$$

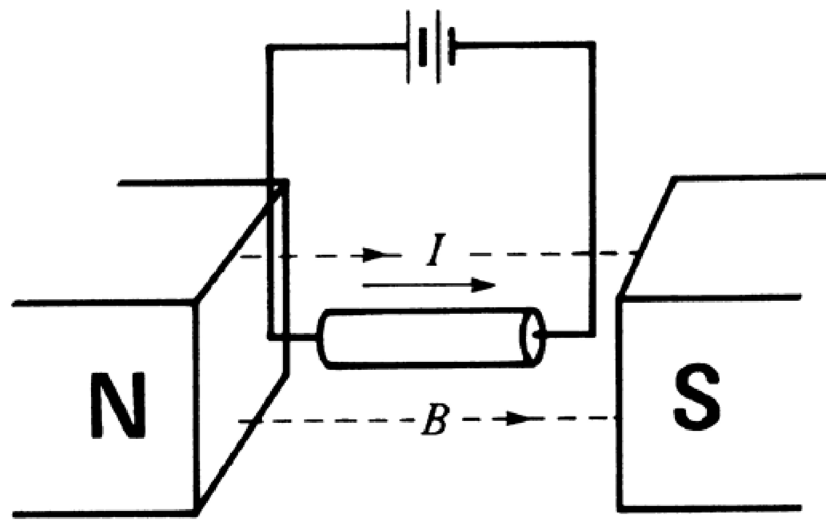
where θ is the angle between the wire and flux density vector. The direction of the force is determined by the right hand rule



Induced force on a current-carrying conductor

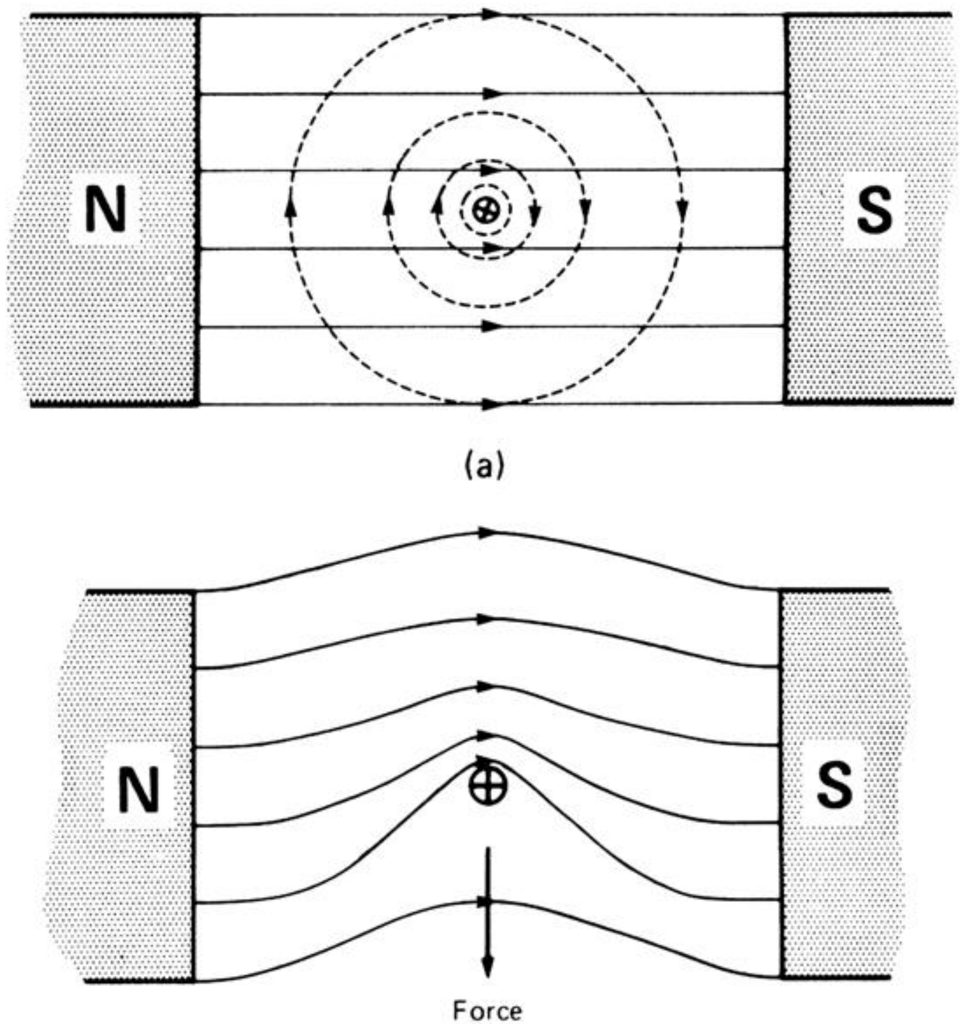


$$\theta = 90\,\mathrm{deg.} \Rightarrow F = BIl$$



$$\theta = 0\,\mathrm{deg.} \Rightarrow F = 0$$

Induced Force on a Current Carrying Conductor



Problems (Chap. 1)

- 5, 6, 8, 10, 12, 14